Reliability Analysis in Geotechnics with Finite Elements — Comparison of Probabilistic, Stochastic and Fuzzy Set Methods

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Abstract

The finite element method is widely used for solving various problems in geotechnical engineering practice. The input parameters required for the calculations are generally imprecise. The paper is devoted to a comparison of probabilistic, stochastic and fuzzy set method for reliability analysis with respect to its applicability for practical problems in geotechnical engineering. Emphasis will be given by comparing the effects of modelling uncertainty using different methods, with special reference to the role of spatial correlation. After introducing some basic notions about the approaches, this article shows that the results obtained with the fuzzy set method for a simple bearing capacity problem agree with the outcomes by a probabilistic and a stochastic method. Advantages and shortcomings of either approach with respect to practical applications will be discussed.

Keywords

finite element method, probabilistic, fuzzy set, stochastic modelling, random field, spatial correlation

1 Introduction

It is well known that material parameters of geomaterials may scatter within a considerable range. Thus, a high degree of uncertainty may be introduced in any type of analysis if material parameters are treated as deterministic values. There is no agreement about what method should be used, to account for these uncertainties especially in practical geotechnical problems where usually not sufficient information is available for a rigorous stochastic analysis, because site investigation and laboratory testing are restricted due to financial und time constraints.

It is still possible to use probabilistic methods in these problems by making suitable assumptions on the statistics of the uncertainties, at least to some extent, by combining different sources of information via Bayes' theorem. However, the numerical values obtained by probabilistic analysis (e.g. probability of failure) are quite sensitive to changes in the input distribution parameters ([1, 13]), but play an important rule in comparative and qualitative studies [14]. On the other hand, Fuzzy set methods provide an appropriate mathematical model which can be used for quantitative assessment.

In the developed methodology point estimate methods (PEM) for probabilistic analyses and fuzzy set method for possibilistic analyses together with a finite element model is used. Emphasis will be given to comparison with methods employing a stochastic model, which means that the parameters are described by spatial random fields (e.g. [7]). This stochastic approach employs the Monte-Carlo method and is used in this paper as a reference.

Both variability and spatial correlation lengths of material properties can affect the reliability of geotechnical systems. In this article, elasto-plastic finite element analysis has been combined with theories mentioned above to investigate the influence of material variability and spatial correlation lengths on the computation of the bearing capacity of a smooth rigid strip footing on a weightless soil with shear strength parameters c and φ under plane strain conditions [14]. The soil stratum is compressed by incrementally displacing the top surface vertically downwards. Geometry and boundary conditions of the problem are shown in figure 1.

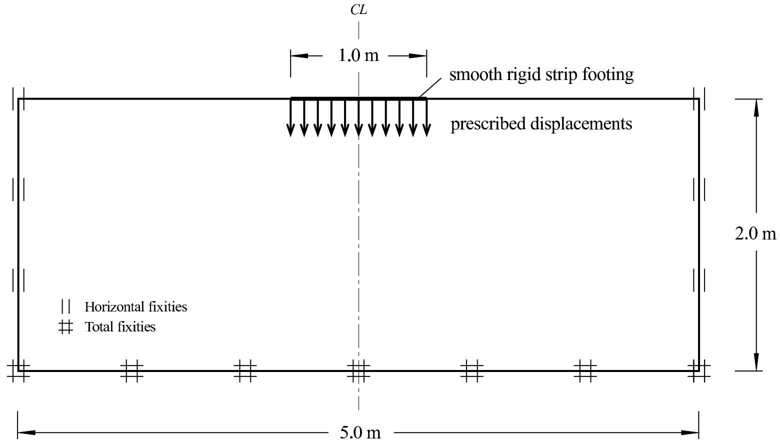

Figure 1: Geometry and boundary conditions

In the simulations, the mean cohesion (μ_c) and mean angle of friction (μ_φ) have been held constant at 100 kN/m^2 and 25° while the coefficient of variation, (COV=σ_c/μ_c), and the spatial correlation length, (Θ), are varied systematically. For this investigation, it is assumed that when the variability in the cohesion is large, the variability in the friction angle will also be large. The material parameters required for the model used are: Young's modulus (E), Poisson's ratio (ν), dilatancy angle (ψ), cohesion (c), and friction angle (φ). In the present study, E, ν and ψ are held constant (at 100.000 kN/m^2, 0.3, and 0, respectively) while c and φ are basic variables. It has to be pointed out that the interaction and cross-correlation between the shear strength parameters is neglected in this study.

The question is how the variability of the shear strength parameters c and φ affects the response given by the dimensionless *bearing capacity factor*, N_c, and consequently the reliability of the structure. The bearing capacity factor is traditionally defined by $N_c = q_f \,/\, c$ where q_f is the computed bearing capacity and c is the cohesion of the soil. The theoretical bearing capacity factor, N_c, for a spatially constant friction angle is given by Sokolovski [19]:

$$N_c = \frac{1}{\tan\varphi}\left[e^{\pi\tan\varphi}\tan^2\left(45+\frac{\varphi}{2}\right)-1\right]$$

2 Spatial variability of soil properties

In principle, the spatial variation of a soil layer can be characterized in detail, but only if a large number of tests can be performed. Thus, for geotechnical purposes a simplification is introduced in which spatial variability is subdivided into two parts, i.e. a linear trend, and a residual variability (stochastic description) about that trend [15]. Figure 2 depicts the value of the soil property, u, at a boring location as a function of depth, z, where $\mu_u(z)$ describes the trend which is represented by a depth-dependent mean value. The stochastic description of the soil property, $u(z)$, consists of the standard deviation, $\sigma_u(z)$, and the scale of fluctuation or autocorrelation length, Θ_u, of $u(z)$.

The spatial correlation length measures the distance within which the property shows relatively strong correlation from point to point. The soil is modelled as a random field ([21, 16]), which is a stochastic process defined by three coordinates in space. This means that the properties of the soil in a specific point are described as a random variable. Rather than a characterization of soil properties at every point, data are used to estimate a smooth trend, and remaining variations are described statistically because of the lack of data.

2.1 Spatial averaging

The mean of large volumes remains the same as the mean of small volumes, but the standard deviation of the average property from one large volume to the next

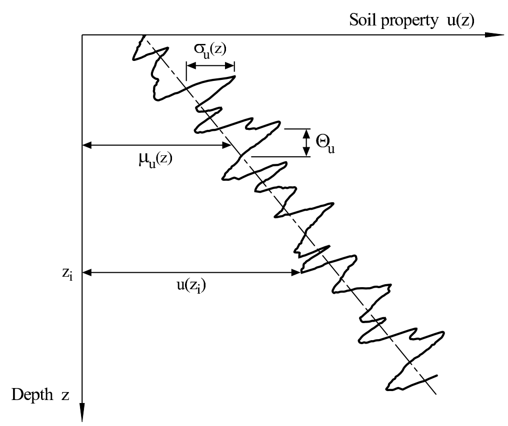

Figure 2: Spatial variability of a soil layer

is smaller than the standard deviation of the average property from one small volume to the next [21]. The extent of averaging of soil properties, $u(z)$, within a large volume depends on the structure of spatial variation. More precisely, the extent of averaging depends on the standard deviation of properties, σ_u, from point to point and on the autocorrelation function. Similarly, the standard deviations of the spatial averages, $u_{\Delta z}$ and u_V, are $\sigma_{u_{\Delta z}}$ and σ_{u_V}, respectively. Therefore, the larger the length Δz or the volume V over which the property is averaged, the more variations of u tends to produce a reduction in the process of spatial averaging. This tends to originate a reduction in standard deviation as the size of the averaged length or volume increases. The so-called reduction factor $\Gamma_u(V)$ is defined as the dimensionless ratio between σ_{u_V} and σ_u ($\Gamma_u(V) = \sigma_{u_V} / \sigma_u$).

The square of the reduction factor, Γ_u^2, will be called the variance function, whereas for the two-dimensional case it will take the form: $\Gamma_u^2(\Delta z) = \Theta_u / \Delta z$ for $\Delta z \geq \Theta_u$. This relationship in fact defines the *scale* Θ_u, and provides a basis for estimating this parameter of $u(z)$ (figure 2). A useful interpretation of this relationship is that Θ_u is the *elementary distance* that can be used to measure Δz. Other assumptions for the determination of this variance reduction factor are presented in e.g. [10, 22].

3 Probabilistic approach

3.1 The point estimate method

An alternative approach for calculating the statistical moments of the limit state function, denoted by G($\boldsymbol{X}$), where $\boldsymbol{X}$ is the collection of random input variables, is the point estimate method (PEM). The method is essentially a weighted average method similar to numerical integration formulas involving *sampling points* and *weighting parameters*. The method seeks to replace a given continuous probability density function, with a discrete function having the same first three central moments (mean value μ, standard deviation σ and skewness ν). The point estimate method is able to account for up to three moments.

The most common point estimate method was developed by Rosenblueth [17]. In addition to Rosenblueth's method, there are many other PEMs developed by various researchers, including the methods of Evans [6], Zhou and Nowak [24], Harr [9] and of Li [11]. In the present study the point estimate methods by Rosenblueth, Harr and Zhou and Nowak are used to obtain the statistical moments of the bearing capacity factor N_c. A brief description of the methods is given below.

PEM by Rosenblueth: Rosenblueth [17] developed a point estimate method which concentrates the probability density of a continuous random variable $\boldsymbol{X}$ into two estimate points. If G($\boldsymbol{X}$) is a function of n basic variables whose skewness is zero but which may be correlated, 2^n points are chosen to include all possible combinations so that the value of each variable is one standard deviation above or below its mean value.

PEM by Harr: In particular the point estimate method by Harr [9] extends Rosenblueth's PEM. Harr proposed an alternative method which starts from the correlation matrix of the data. This matrix is a real symmetric matrix of order n, the number of random variables which can be diagonalized by an orthogonal eigenvector matrix. The correlation matrix can be represented by a hypersphere of radius $\sqrt{n}$ centered at the corresponding expected values of x_n in the standardized coordinate system. The eigenvector starts from the origin of expected values in their respective directions and each eigenvector intersects the sphere at two points. These points of intersections provide the $2n$ point estimates for calculating the statistical moments of G($\boldsymbol{X}$).

PEM by Zhou and Nowak: In the approach proposed by Zhou and Nowak [24] predetermined points in the standard normal space are used to compute the statistical parameters of a function of multiple random variables $\boldsymbol{X}$. These points must be transformed in the typically correlated and non standard normal distributed space. The integration of G($\boldsymbol{X}$) can be achieved using a non-product formula. Zhou and Nowak provide a set of numerical integration formulas. In this work the $2n^2$+1 formula (ZN III) is used which leads to $2n^2$+1 realizations of G($\boldsymbol{X}$).

3.2 Stochastic modelling of soil properties

The finite element code [2] used in the proposed approach to calculate the bearing capacity q_f, require the soil profile to be modelled using homogeneous layers with constant soil properties. For this reason soil properties have to be defined not only for a certain point in space, but also for the entire domain which is used in the calculation process. Due to the fact of spatial averaging of soil properties the coefficient of variation is reduced significantly as described above. In this study, the variance reduction factor Γ by Vanmarcke [22] is used and can be obtained by

$$\Gamma^2 = \left[\frac{\Theta}{L_u}\left(1 - \frac{\Theta}{4L_u}\right)\right]$$

for $\Theta/L_u \leq 2$, where Θ is the autocorrelation length and L_u is the length of the potential failure surface. For $\mu_\varphi = 25°$ the length of the failure surface L_u yields a value of approximately 10.5 m.

4 Stochastic approach

The model of Fenton and Griffiths [7] combines random field theory with an elasto-plastic finite element algorithm in a Monte-Carlo framework (RFEM). The spatially varying and cross-correlated random fields are generated using the so-called Local Average Subdivision (LAS) method which produces local arithmetic averages of the lognormally distributed random field over each element. Thus, each element is assigned a random value of ln c (c is the soil cohesion) as a local average, over the element size, of the continuously varying random field having point statistics. The element values thus correctly reflect the variance reduction due to arithmetic averaging over the element as well as the cross-correlation structure dictated by spatial correlation length, $\Theta_{ln\,c}$. For the correlation structure of the underlying generated fields an exponentially decaying isotropic correlation function is assumed, $\rho(\tau) = \exp(-2\tau / \Theta_{ln\,c})$ where τ is the absolute distance between any two points in the field. A typical deformed finite element mesh at failure is shown in figure 3. Lighter regions in the illustration indicate stronger material and darker regions indicate weaker material, which have triggered quite irregular failure mechanisms.

The soil cohesion, c, is assumed to be lognormally distributed with mean μ_c, standard deviation σ_c, and spatial correlation length $\Theta_{ln\,c}$. For the friction angle, φ, a bounded distribution is selected. For each set of statistical properties given in Table 1 according to [7], Monte-Carlo simulations have been performed, which involves 1000 repetitions of the soil property random fields and the subsequent finite element analysis. A different value for the bearing capacity, and after normalization by the mean cohesion μ_c, a different value for the bearing capacity factor, N_{ci}, is obtained for each of the n Monte-Carlo simulations by $N_{ci} = q_{fi} / \mu_c$, i = 1,2,...,n. These values are then analysed statistically leading to an expected value E[N_c], and standard deviation, s[N_c].

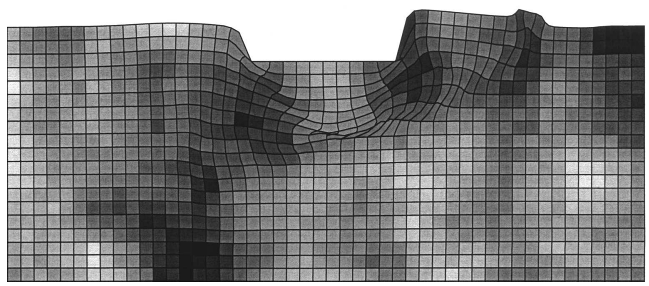

Figure 3: Typical deformed finite element mesh at failure from [7]

5 Fuzzy set approach

Zadeh [23] used the theory of fuzzy sets as a basis for possibility to model uncertainties. Although possibility distributions seem to be similar to probability distributions, possibility calculus, which is used to derive the membership function of the performance of a system from the membership functions of the uncertain variables, is fundamentally different than probability calculus. The main difference between the axioms of possibility and probability measures is that the possibility of a union of events (disjoint or not) is equal to the maximum of the possibilities of the individual events, whereas the probability of a union of disjoint events is equal to the sum of the probabilities of these events (see e.g. discussion in [4]). Therefore, fuzzy set approach is an alternative to probability.

5.1 Fuzzy numbers

$\mathbb{F}(X)$ denotes the collection of fuzzy subsets of a set X. A fuzzy set $A \in \mathbb{F}(X)$ is characterized by (and can be identified with) its membership function $m_A(x)$, $0 \leq m_A(x) \leq 1$, describing the degree of possibility that the variable A takes the value x of X. The fuzzy sets $[A]^\alpha = x \in X : m_A(x) \geq \alpha$ are the so-called α-level sets of A, i.e. the variable A fluctuates in the range $[A]^\alpha$ with possibility degree α. Given a function $f\colon X \rightarrow Y$, the extension principles by Zadeh [23] allows to extend it to a function $f\colon\mathbb{F}(X) \rightarrow \mathbb{F}(Y)$ by $m_{f(A)}(y) = \sup\{m_A(x), x \in f^{-1}(y)\}$.

$A \in \mathbb{F}\left(\mathbb{R}^{\mathrm{d}}\right)$ is called a fuzzy vector, if each of its α-level sets is convex and compact ($0 < \alpha < 1$), and $[A]^1$ contains exactly one point. In the case of $d = 1$, A is referred to as a fuzzy number. If $f\colon \mathbb{R}^{\mathrm{d}} \rightarrow \mathbb{R}$ is continuous and A a fuzzy vector, the function value $f(A)$ is a fuzzy number, whose level sets are computed by set theoretic evaluation: $[f(A)]^\alpha = f([A]^\alpha)$.

5.2 Fuzzification Method

Dubois and Prade [5] have proposed methods, which are based on judgement and/or on statistical data but there is no commonly accepted procedure for estimating the possibility distribution of a variable. To compare probabilistic and fuzzy set-based methods, we first construct a fuzzy set of an uncertain variable on the basis of a given probability distribution by means of the *least conservative principle* [12]. In this way, we ensure that both models are constructed using the same data. In this paper, the principle is applied to construct a fuzzy set on the basis of a given lognormal probability distribution in such a way that the range between the 5% and the 95%-fractile represents the support (the upper and lower bound value corresponds to $\alpha = 0$) of the triangular fuzzy number where the ultimate value, the core, respectively is at the modal value, which is the most frequent value (figure 4 and 5). Since the data is based on the lognormal distributions according to section 3.3, it has to be pointed out that autocorrelation is considered already.

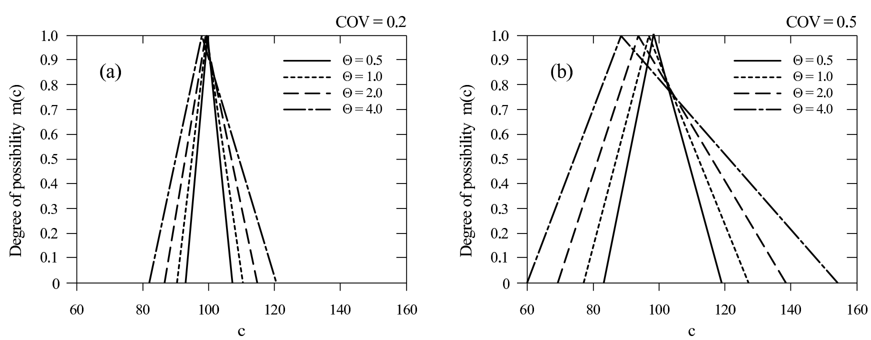

Figure 4: Fuzzy input parameter c, for a) COV of 0.2 and b) COV of 0.5

5.3 Fuzzy finite element analysis

When the input variables are defined as fuzzy numbers, the computation of the fuzzy response quantity has to be performed. This is achieved by constructing a possibility distribution for the response quantity which is based on the extension principle mentioned above. The principle relates the possibility distribution of fuzzy input variables to the possibility distribution of the fuzzy response function, whereas the α-level concept is used to numerically implement the extension principle. In this approach, the fuzzy function is a finite element model that transforms input fuzzy data to a desired fuzzy output quantity. By replacing the fuzzy numbers in the solution model with intervals, the fuzzy computation reduces to a series of interval analyses, where the minimum and the maximum of the 2^n values

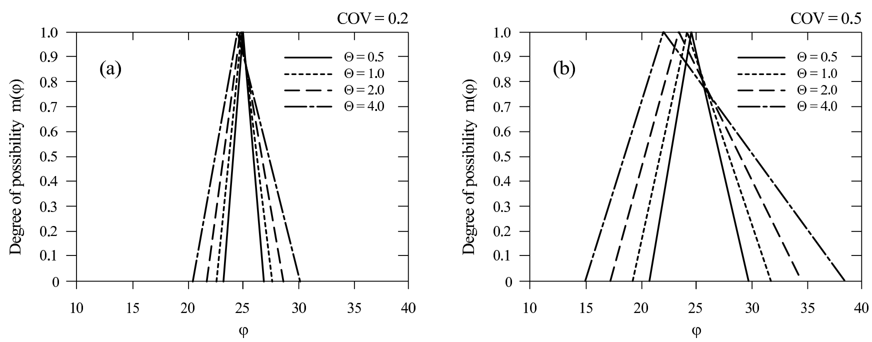

Figure 5: Fuzzy input parameter φ, for a) COV of 0.2 and b) COV of 0.5

define the resulting interval (n is the number of fuzzy input variables). Repeating this process for all selected α-levels, a set of resulting intervals corresponding to the selected α-levels is obtained and define the final output, the response membership function of the dimensionless bearing capacity factor, N_c (figure 6). The higher the number of α-levels under consideration, the greater the accuracy of the possibility distribution of the response. The total number of finite element runs that is involved is $N \cdot 2^n$, where N is the number of α-levels.

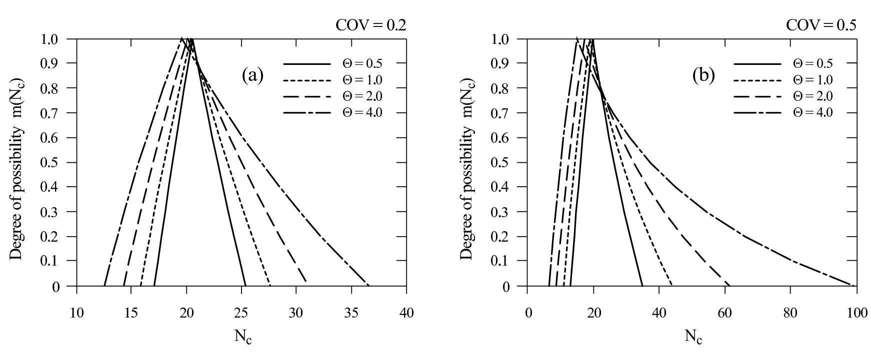

Figure 6: Possibility distribution of the bearing capacity factor, N_c, for a) COV of 0.2 and b) COV of 0.5

5.4 Defuzzification Method

For defuzzification a method based on weighted possibilistic mean and variance of fuzzy numbers is used in this paper. Carlsson and Fuller [3] suggested the notations of weighted possibilistic mean value and variance of fuzzy numbers, which are consistent with the extension principle. Furthermore, they showed that the weighted variance of linear combinations of fuzzy numbers can be computed in a similar manner as in probability theory:

$$\mathrm{E}\left[X^r\right] = \sum_{i=1}^{N} \frac{\alpha_i . x^r_{\alpha_i}}{N}$$

with $x^r_{\alpha_i}$ = 1/2 ($x^r_{\alpha_{i,L}} + x^r_{\alpha_{i,U}}$), where E[X^r] represents the level-weighted r^{th} moment of all α-level sets. α_i denotes the α-level, N the number of α-levels considered and $x^r_{\alpha_i}$ the arithmetic means of all α-level sets, that is, the weight of the arithmetic mean of $x^r_{\alpha_{i,L}}$ and $x_{\alpha_{i,U}}$ is just α.

6 Results and discussion

Figure 7 depicts the influence of Θ and COV$_c$ on the sample coefficient of variation of the estimated bearing capacity factor, COV$_{N_c}$ = s$_{N_c}$/E[N_c] computed by the random field model (RFEM) and by using the probabilistic and the fuzzy set approach.

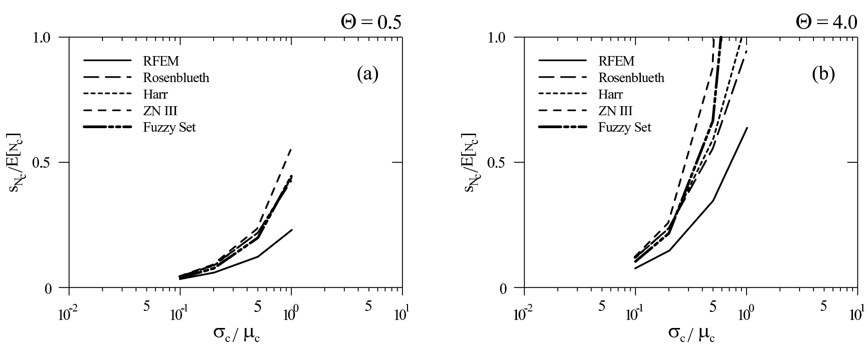

Figure 7: Coefficient of variation of N_c, a) Θ=0.5 and b) Θ=4.0

To have an assessment on the performance of all the approaches, the results from the fuzzy solution are also included in those plots. The figure shows how the bearing capacity factor varies with soil variability, and the spatial correlation length. The plots indicate that COV$_{N_c}$ is positively correlated with both COV$_c$ and Θ, i.e. the variability in E[N_c] increases with the variability in the soil (the higher the spatial correlation length the higher the increase). The results compare well with the COV$_{N_c}$ by the random field method, which represents a more sophisticated method. The PEM methods as well as the fuzzy set method capture the overall behaviour of the analysed ratio and is fairly accurate for moderate magnitudes of the variability in the soil, i.e. COV$<$0.5.

Dubois and Prade [5] have shown that a possibility distribution (fuzzy set A) constructed starting from few statistical data may be used to represent a wide class of probability distributions (compatible with the available information) and to consistently define upper and lower probability distributions, $F_L(x)$ and $F_U(x)$. These bounds may be rewritten in terms of the membership function of the fuzzy set A as $F_L(x) = \sup\{m_A(x), x \leq \omega\}$ and $F_U(x) = \inf\{1\text{-}m_A(x), x > \omega\}$, where ω describes the value x with the degree of possibility, $m_A(x)$=1 [8].

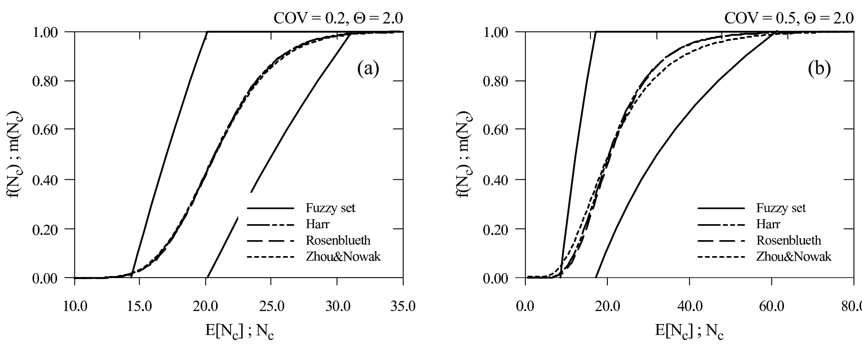

Figure 8: Cumulative distribution functions of E[N_c] assumed as lognormally distributed and membership function of N_c, for a) COV of 0.2 and b) COV of 0.5

Figure 8 shows the possibility and probability of the bearing capacity factor N_c. It can be seen that the possibility is always greater than the probability. Also note that, for this case, the possibility is 1.0 when the probability is 0.5. These results are in line with other studies, e.g. Smith et al. [18] showed that if the fuzzy membership function for a random variable is based on the mean and standard deviation of a probabilistic random variable, the possibility of failure is one when the probability of failure is fifty-percent. Therefore, fuzzy set theory may be used to obtain conservative bounds for probability [13].

From a practical point of view, it would be of interest to estimate the probability of *design* failure [7], defined here as occurring when the computed bearing capacity factor, N_c, is less than the deterministic value based on the mean angle of friction divided by a factor of safety F, i.e. 20.7/F (the mean angle of friction φ = μ_φ = 25 degrees, then the deterministic value of N_c yields approximately 20.7).

With the obtained mean value and standard deviation of the performance function based on the PEM assuming a lognormal distribution the probability of design failure (P[N_c<20.7/F]) can be evaluated. For the case where Θ=4.0 figure 9 compares the probability of design failure for two different factors of safety F obtained by probabilistic methods and random field method [14]. The results indicate that the higher the variability (COV) the higher the probability of design failure and show that the proposed method predicts the basic behaviour of relatively simple functions of random variables, but the accuracy is significantly reduced for large coefficients of variation of the input variables.

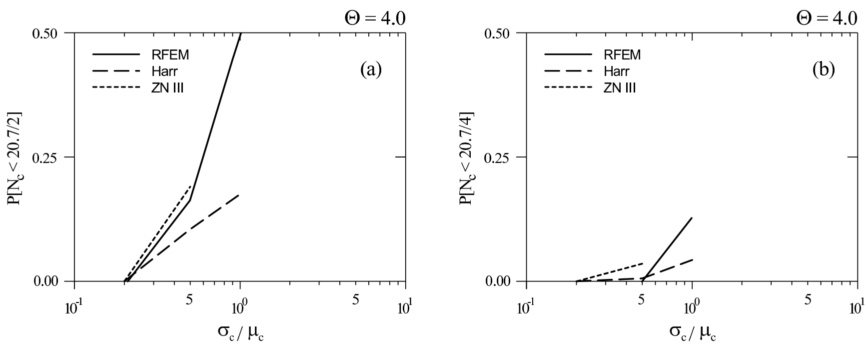

Figure 9: Probabilistic and stochastic approach with Θ=4.0: Influence of factor of safety for a) F=2 and b) F=4

In order to determine a possibility of design failure the membership functions for the response bearing capacity factor, N_c, are compared with the allowable responses, i.e. 20.7/F as already mentioned. Figure 10 illustrates how the possibility of design failure varies as a function of COV_{N_c} and the ratio of the target value 20.7/F. The fuzzy set method also captures the basic behaviour in terms of the possibility of design failure for the given problem. The outcomes show that the higher the variability (COV) the higher the possibility of design failure. Similar observations can be made about the relations between possibility and probability as described by figure 8, i.e. that the possibility of failure is one when the probability of failure is fifty-percent. However, Stroud et al. [20] reported that even though the possibility of failure was always greater than the probability of failure for a particular problem with two failure modes, the assumption that possibilistic design is conservative is not a valid assumption when there are many failure modes.

7 Concluding remarks

The general objective of this paper is to study the differences between probabilistic, stochastic and fuzzy set methods for modelling uncertainties with respect to a simple practical problem for geotechnical engineering. It is argued that the uncertainties associated with material and model parameters are covered in a rational way in the probabilistic and fuzzy set approach. The true probability distributions

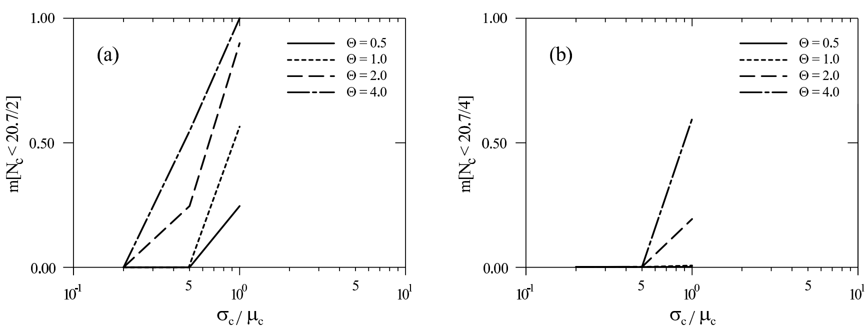

Figure 10: Fuzzy set approach: Influence of factor of safety for a) F=2 and b) F=4

of the uncertain soil parameters, c (cohesion), and, φ (friction angle), are used as the scale to compare the probabilistic and fuzzy set based methods. Generally speaking the outcome of point estimate methods and the fuzzy set method agreed reasonably well with the results obtained by the random field method. An advantage of the fuzzy set approach, from a practical point of view, is the determination of an upper and lower bound to the probability in an efficient way. The results are in line with other studies, even for membership functions as simple as the triangular functions employed here. For the given system and the given data about uncertainties, probabilistic and stochastic analysis yields the probability of failure and fuzzy set analysis yields the possibility of failure, which also varies between zero and one. However, the two measures are not directly comparable, but the results considered were intended to be of easy comprehension and to allow the establishment of a comparison and a correspondence between the methods.

It is acknowledged that the comparisons presented are not rigorous in a mathematical sense and the authors are aware of the discussion on whether the assumptions made in these methods allow a comparison at all. However, from a practical point of view this type of *uncertainty* can be accepted, because it is a significant step forward to be able to account for uncertainties in material parameters using high level numerical methods and keeping the computational effort acceptable. In practice there will always be a trade off between mathematical rigour and practical benefits achievable, which is true in particular in geotechnical engineering. The work presented here should be seen as a step towards a more realistic modelling in geotechnical engineering by demonstrating the applicability of various approaches and should not be seen as a recommendation for one or the other method, at least not at the present stage of developments.

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